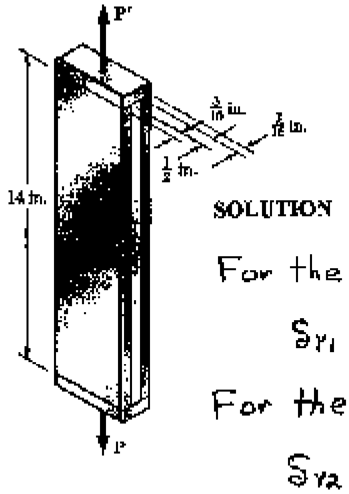
PROBLEM 2.116

2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

*2.116 For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and is then decreased back to zero.



SOLUTION

For the mild steel $\quad A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$

$$\delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(14)(50\times10^3)}{29\times10^6} = 0.024138 \text{ in}$$

For the tempered steel $\quad A_2 = 2(\frac{3}{16})(2) = 0.75 \text{ in}^2$

$$\delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(14)(100\times10^3)}{29\times10^6} = 0.048276 \text{ in.}$$

Total area: $\quad A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$ The mild steel yields. Tempered steel is elastic.

Forces $\quad P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3$ lb

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times10^6)(0.75)(0.04)}{14} = 62.14\times10^3 \text{ lb.}$$

Stresses $\quad \sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50\times10^3$ psi

$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14\times10^3}{0.75} = 82.86\times10^3 \text{ psi}$$

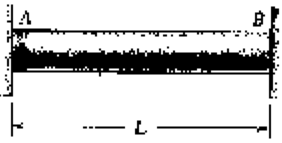
Unloading $\quad \sigma' = \frac{P}{A} = \frac{112.14}{1.75} = 64.08\times10^3$ psi

Residual stresses

$$\sigma_{1,res} = \sigma_1 - \sigma' = 50\times10^3 - 64.08\times10^3 = -14.08\times10^3 \text{ psi} = -14.08 \text{ ksi}$$

$$\sigma_{2,res} = \sigma_2 - \sigma' = 82.86\times10^3 - 64.08\times10^3 = 18.78\times10^3 \text{ psi} = 18.78 \text{ ksi}$$

PROBLEM 2.117



2.117 A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of 8°C. The steel is assumed to be elastoplastic with $\sigma_Y = 250$ MPa and $E = 200$ GPa. Knowing that $\alpha = 11.7 \times 10^{-6}/°C$, determine the stress in the bar (*a*) when the temperature is raised to 165°C, (*b*) after the temperature has returned to 8°C.

SOLUTION

Determine temperature change to cause yielding

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_Y L}{E} + L\alpha(\Delta T)_Y = 0$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E\alpha} = \frac{250 \times 10^6}{(200 \times 10^9)(11.7 \times 10^{-6})} = 106.838\ °C$$

But $\Delta T = 165 - 8 = 157\ °C$

(a) Yielding occurs $= \quad \sigma = -\sigma_Y = -250$ MPa

Cooling $(\Delta T)' = 157°C$

$$\delta' = \delta_P' + \delta_T' = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)'$$

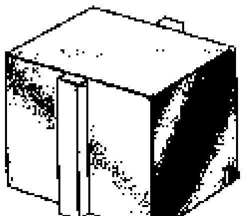
$$= -(200 \times 10^9)(11.7 \times 10^{-6})(157) = -367.38 \times 10^6 \text{ Pa}$$

(b) $\sigma_{res} = -\sigma_Y + \sigma' = -250 \times 10^6 + 367.38 \times 10^6 = 117.38 \times 10^6$ Pa

$= 117.4$ MPa

PROBLEM 2.118

2.118 A narrow bar of aluminum is bonded to the side of a thick steel plate as shown. Initially, at $T_1 = 20°C$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1, determine (*a*) the highest temperature T_2 that does *not* result in residual stresses, (*b*) the temperature T_2 that will result in a residual stress in the aluminum equal to 100 MPa. Assume $\alpha_a = 23.6 \times 10^{-6}/°C$ for the aluminum and $\alpha_s = 11.7 \times 10^{-6}/°C$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 70$ GPa and $\sigma_Y = 100$ MPa. (*Hint:* Neglect the small stresses in the plate.)



SOLUTION

Determine temperature change to cause yielding

$$\delta = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T_Y) = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{100 \times 10^6}{(70 \times 10^9)(23.6 - 11.7)(10^{-6})} = 120.04\,°C$$

(a) $T_{2Y} = T_1 + (\Delta T)_Y = 20 + 120.04 = 140.04\,°C$

After yielding

$$\delta = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling

$$\delta' = \frac{P'L}{AE} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{res} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

Set $\sigma_{res} = -\sigma_Y$

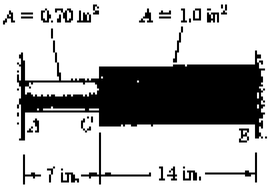
$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\Delta T = \frac{2\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{(2)(100 \times 10^6)}{(70 \times 10^9)(23.6 - 11.7)(10^{-6})} = 240.1\,°C$$

(b) $T_2 = T_1 + \Delta T = 20 + 240.1 = 260.1\,°C$

If $T_2 > 260.1\,°C$, the aluminum bar will most likely yield in compression.

PROBLEM 2.119

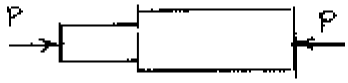


2.119 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 38°F. The steel is assumed elastoplastic, with $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. The temperature of both portions of the rod is then raised to 250 °F. Knowing that $\alpha = 6.5 \times 10^{-6}/°F$, determine (*a*) the stress in portion *AC*, (*b*) the deflection of point *C*.

SOLUTION

$$\delta_{B/A} = \delta_{B/A,P} + \delta_{B/A,T} = 0 \qquad \text{(constraint)}$$

Determine ΔT to cause yielding in AC.



$$-\frac{P L_{AC}}{E A_{AC}} - \frac{P L_{CB}}{E A_{CB}} + L_{AB}\,\alpha(\Delta T) = 0$$

$$(\Delta T) = \frac{P}{L_{AB} E \alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding $\quad P = A_{AC}\sigma_Y$

$$(\Delta T)_Y = \frac{A_{AC}\sigma_Y}{L_{AB} E \alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) = \frac{(0.70)(36\times10^3)}{(21)(29\times10^6)(6.5\times10^{-6})}\left(\frac{7}{0.70} + \frac{14}{1.0}\right)$$

$$= 152.785\ °F$$

Actual $\Delta T = 250 - 38 = 212\,°F > (\Delta T)_Y \therefore$ yielding occurs.

$$\sigma_{AC} = -\sigma_Y = -36 \text{ ksi}$$

$$P = \sigma_Y A_{AC} = (36\times10^3)(0.70) = 25.2\times10^3 \text{ lb}$$

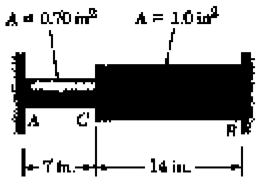
$$\delta_C = -\delta_{C/B} = \frac{P L_{CB}}{E A_{CB}} - L_{CB}\,\alpha(\Delta T)$$

$$= \frac{(25.2\times10^3)(14)}{(29\times10^6)(1.0)} - (14)(6.5\times10^{-6})(212)$$

$$= 0.012166 - 0.019292 \qquad = -0.007126 \text{ in}$$

$$\delta_C = 0.00712 \text{ in} \leftarrow$$

PROBLEM 2.120

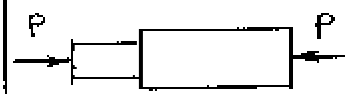


2.119 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 38°F. The steel is assumed elastoplastic, with $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. The temperature of both portions of the rod is then raised to 250 °F. Knowing that $\alpha = 6.5 \times 10^{-6}/°F$, determine (*a*) the stress in portion *AC*, (*b*) the deflection of point *C*.

***2.120** Solve Prob. 2.119, assuming that the temperature of the rod is raised to 250 °F and then returned to 38 °F.

SOLUTION

$\delta_{B/A} = \delta_{B/A,P} + \delta_{B/A,T} = 0$ (constraint)



Determine ΔT to cause yielding in AC.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} + L_{AB}\alpha(\Delta T) = 0$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) = \frac{P}{(21)(29\times10^6)(6.5\times10^{-6})}\left(\frac{7}{0.70} + \frac{14}{1.0}\right)$$

$= 6.0629\times10^{-3}\,P$ At yielding $P_Y = \sigma_Y A_{AC} = (36\times10^3)(0.7) = 25.2\times10^3$ lb

$(\Delta T)_Y = (6.0629\times10^{-3})(25.2\times10^3) = 152.785$ °F

Actual $\Delta T = 250 - 38 = 212$ °F $> (\Delta T)_Y$ $\therefore$ yielding occurs.

$\sigma_{AC} = -\sigma_Y = -36\times10^3$ psi

$$\delta_C = -\delta_{B/C} = \frac{PL_{CB}}{EA_{CB}} - L_{BC}\alpha(\Delta T) = \frac{(25.2\times10^3)(14)}{(29\times10^6)(1.0)} - (14)(6.5\times10^{-6})(212)$$

$= 0.012176 - 0.019292 = -0.007116$ in

Cooling $\Delta T' = 212$ °F $P' = \frac{\Delta T}{6.0629\times10^{-3}} = \frac{212}{6.0629\times10^{-3}}$

$$P' = \frac{\Delta T'}{6.0629\times10^{-3}} = \frac{212}{6.0629\times10^{-3}} = 34.967\times10^3 \text{ lb.}$$

(a) Residual stress in AC

$$\sigma_{AC,res} = -\sigma_Y + \frac{P'}{A_{AC}} = -36\times10^3 + \frac{34.967\times10^3}{0.7} = 13.95\times10^3 \text{ psi}$$

$= 13.95$ ksi

$$\delta_C' = -\delta_{B/C}' = -\frac{P'L_{CB}}{EA_{CB}} + L_{CB}\alpha(\Delta T)$$

$$= -\frac{(34.967\times10^3)(14)}{(29\times10^6)(1.0)} + (14)(6.5\times10^{-6})(212)$$

$= -0.016881 + 0.019292 = 0.002411$ in

$\delta_{C,P} = \delta_C + \delta_C' = -0.007116 + 0.002411 = -0.00471$ in

0.00471 in ←