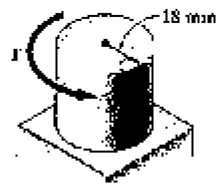
CHAPTER 3

PROBLEM 3.1



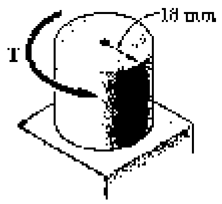
3.1 Determine the torque T which causes a maximum shearing stress of 70 MPa in the steel cylindrical shaft shown.

SOLUTION

$$\tau_{max} = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^4$$

$$T = \frac{\pi}{2}c^3\tau_{max} = \frac{\pi}{2}(0.018)^3(70\times10^6) = 641\ \text{N}\cdot\text{m}$$

PROBLEM 3.2



3.2 Determine the maximum shearing stress caused by a torque of magnitude $T = 800$ N·m.

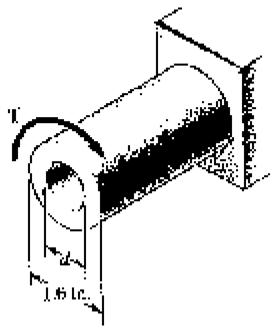
SOLUTION

$$\tau_{max} = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^4$$

$$\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(800)}{\pi(0.018)^3} = 87.3\times10^6\ \text{Pa}$$

$$87.3\ \text{MPa}$$

PROBLEM 3.3



3.3 Knowing that the internal diameter of the hollow shaft shown is $d = 0.9$ in., determine the maximum shearing stress caused by a torque of magnitude $T = 9$ kip·in.

SOLUTION

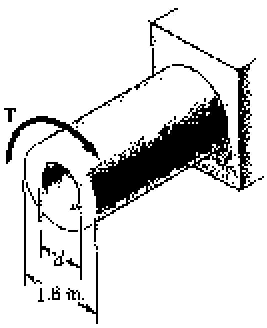
$$c_2 = \tfrac{1}{2}d_2 = (\tfrac{1}{2})(1.6) = 0.8\ \text{in.} \qquad c = 0.8\ \text{in.}$$

$$c_1 = \tfrac{1}{2}d_1 = (\tfrac{1}{2})(0.9) = 0.45\ \text{in}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.8^4 - 0.45^4) = 0.5790\ \text{in}^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{(9)(0.8)}{0.5790} = 12.44\ \text{ksi}$$

PROBLEM 3.4



3.4 Knowing that $d = 1.2$ in., determine the torque T which causes a maximum shearing stress of 7.5 ksi in the hollow shaft shown.

SOLUTION

$$c_2 = \tfrac{1}{2}d_2 = (\tfrac{1}{2})(1.6) = 0.8\ \text{in} \qquad c = 0.8\ \text{in}$$

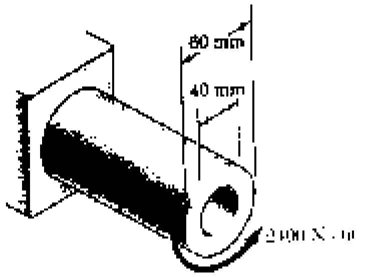
$$c_1 = \tfrac{1}{2}d_1 = (\tfrac{1}{2})(1.2) = 0.6\ \text{in}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.8^4 - 0.6^4) = 0.4398\ \text{in}^4$$

$$\tau_{max} = \frac{Tc}{J}$$

$$T = \frac{J\tau_{max}}{c} = \frac{(0.4398)(7.5)}{0.8} = 4.12\ \text{kip}\cdot\text{in}$$

PROBLEM 3.5



3.5 (*a*) For the hollow shaft and loading shown, determine the maximum shearing stress. (*b*) Determine the diameter of a solid shaft for which the maximum shearing stress is the same as in part *a*.

SOLUTION

$c_1 = \frac{1}{2} d_1 = (\frac{1}{2})(0.040) = 0.020 \text{ m}$

$c_2 = \frac{1}{2} d_2 = (\frac{1}{2})(0.060) = 0.030 \text{ m} \qquad c = 0.030 \text{ m}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.030^4 - 0.020)^4$

$= 1.0210 \times 10^{-6} \text{ m}^4$

(a) $\tau_{max} = \frac{Tc}{J} = \frac{(2400)(0.03)}{1.0210 \times 10^{-6}} = 70.52 \times 10^6 \text{ Pa}$

70.5 MPa ◄

(b) $\tau = \frac{Tc_3}{J}, \quad J = \frac{\pi}{2} c_3^4 \qquad \tau = \frac{2T}{\pi c_3^3}$

$c_3^3 = \frac{2T}{\pi \tau} = \frac{(2)(2400)}{\pi(70.52 \times 10^6)} = 21.67 \times 10^{-6} \text{ m}^3$

$c_3 = 27.88 \times 10^{-3} \text{ m} \qquad d_3 = 2c_3 = 55.8 \times 10^{-3} \text{ m} \qquad 55.8 \text{ mm}$ ◄

PROBLEM 3.6

3.6 (*a*) Determine the torque which may be applied to a solid shaft of 90-mm outer diameter without exceeding an allowable shearing stress of 75 MPa. (*b*) Solve part *a*, assuming that the solid shaft is replaced by a hollow shaft of the same mass and of 90-mm inner diameter.

SOLUTION

(a) For the solid shaft $\quad c = \frac{1}{2} d = (\frac{1}{2})(0.090) = 0.045 \text{ m}$

$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\pi}{2}(0.045)^3 = 143.14 \times 10^{-6} \text{ m}^3$

$\tau_{max} = \frac{Tc}{J} \quad \therefore \quad T = \frac{\tau_{max} J}{c} = (75 \times 10^6)(143.14 \times 10^{-6}) = 10.74 \times 10^3 \text{ N}\cdot\text{m}$

$10.74 \text{ kN}\cdot\text{m}$ ◄

(b) Hollow shaft $\qquad c_1 = \frac{1}{2} d_1 = (\frac{1}{2})(0.090) = 0.045 \text{ m}$

For equal masses the cross sectional areas must be equal

$A = \pi c^2 = \pi(c_2^2 - c_1^2) \quad \text{or} \quad c_2 = \sqrt{c_1^2 + c^2}$

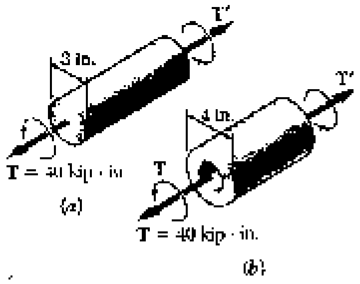
$c_2 = \sqrt{0.045^2 + 0.045^2} = 0.0636396 \text{ m}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 19.3237 \times 10^{-6} \text{ m}^4$

$T = \frac{\tau_{max} J}{c_2} = \frac{(75 \times 10^6)(19.3237 \times 10^{-6})}{0.0636396} = 22.77 \times 10^3 \text{ N}\cdot\text{m}$

$22.8 \text{ kN}\cdot\text{m}$ ◄

PROBLEM 3.7



3.7 (*a*) For the 3-in.-diameter solid cylinder and loading shown, determine the maximum shearing stress. (*b*) Determine the inner diameter of the hollow cylinder, of 4-in. outer diameter, for which the maximum stress is the same as in part *a*.

SOLUTION

(a) Solid shaft $c = \frac{1}{2}d = \frac{1}{2}(3.0) = 1.5$ in.

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(40)}{\pi(1.5)^3} = 7.545 \text{ ksi}$$

(b) Hollow shaft $c_2 = \frac{1}{2}d = \frac{1}{2}(4.0) = 2.0$ in.

$$\frac{J}{c_2} = \frac{\frac{\pi}{2}(c_2^4 - c_1^4)}{c_2} = \frac{T}{\tau_{max}}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau_{max}} = 2.0^4 - \frac{(2)(40)(2.0)}{\pi(7.545)} = 9.25 \text{ in}^4$$

$c_1 = 1.74395$ in $\qquad d_1 = 2c_1 = 3.49$ in

PROBLEM 3.8

3.8 (*a*) Determine the torque which may be applied to a solid shaft of 0.75-in. diameter without exceeding an allowable shearing stress of 10 ksi. (*b*) Solve part *a*, assuming that the solid shaft has been replaced by a hollow shaft of the same cross-sectional area and with an inner diameter equal to half its outer diameter.

SOLUTION

(a) Solid shaft: $c = \frac{1}{2} = (\frac{1}{2})(0.75) = 0.375$ in.

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.375)^4 = 0.031063 \text{ in}^4 \qquad \tau_{max} = 10$ ksi

$$T = \frac{J\tau_{max}}{c} = \frac{(0.031063)(10)}{0.375} = 0.828 \text{ kip·in or } 828 \text{ lb·in}$$

(b) Hollow shaft

For the same area as the solid shaft

$$A = \pi(c_2^2 - c_1^2) = \pi\left[c_2^2 - (\tfrac{1}{2}c_2)^2\right] = \tfrac{3}{4}\pi c_2^2 = \pi c^2$$

$$c_2 = \frac{2}{\sqrt{3}}c = \frac{2}{\sqrt{3}}(0.375) = 0.433013 \text{ in}$$

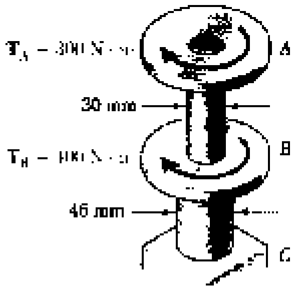
$$c_1 = \tfrac{1}{2}c_2 = 0.216506$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.433013^4 - 0.216506^4) = 0.051772 \text{ in}^4$$

$$T = \frac{\tau_{max}J}{c_2} = \frac{(10)(0.051772)}{0.433013} = 1.196 \text{ kip·in or } 1196 \text{ in·lb}$$

PROBLEM 3.9

3.9 The torques shown are exerted on pulleys *A* and *B*. Knowing that each shaft is solid, determine the maximum shearing stress (*a*) in shaft *AB*, (*b*) in shaft *BC*.



SOLUTION

Shaft AB: $T_{AB} = 300\ \text{N·m}$, $d = 0.030\ \text{m}$, $c = 0.015\ \text{m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi (0.015)^3}$$

$$= 56.588 \times 10^6\ \text{Pa} = \qquad 56.6\ \text{MPa}$$

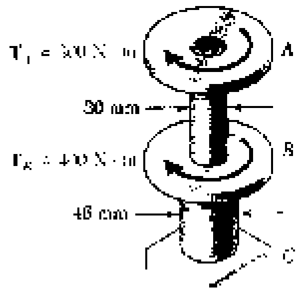
Shaft BC: $T_{BC} = 300 + 400 = 700\ \text{N-m}$

$d = 0.046\ \text{m}$, $c = 0.023\ \text{m}$ $\qquad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi (0.023)^3}$

$$= 36.626 \times 10^6\ \text{Pa} \qquad 36.6\ \text{MPa}$$

PROBLEM 3.10

3.10 The torques shown are exerted on pulleys *A* and *B* which are attached to solid circular shafts *AB* and *BC*. In order to reduce the total mass of the assembly, determine the smallest diameter of shaft *BC* for which the largest shearing stress in the assembly is not increased.



SOLUTION

Shaft AB: $T_{AB} = 300\ \text{N·m}$, $d = 0.030\ \text{m}$, $c = 0.015\ \text{m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi (0.015)^3}$$

$$= 56.588 \times 10^6\ \text{Pa} \qquad 56.6\ \text{MPa}$$

Shaft BC: $T_{BC} = 300 + 400 = 700\ \text{N·m}$

$d = 0.046\ \text{m}$, $c = 0.23\ \text{m}$ $\qquad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi (0.023)^3}$

$$= 36.626 \times 10^6\ \text{Pa} \qquad 36.6\ \text{MPa}$$

The largest stress (56.588×10^6 Pa) occurs in portion AB

Reduce the diameter of BC to provide the same stress.

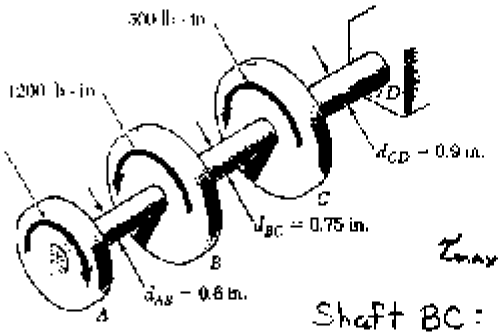
$T_{BC} = 700\ \text{N·m} \qquad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$c^3 = \frac{2T}{\pi \tau_{max}} = \frac{(2)(700)}{\pi (56.588 \times 10^6)} = 7.875 \times 10^{-6}\ \text{m}^3$$

$c = 19.895 \times 10^{-3}\ \text{m} \qquad d = 2c = 39.79 \times 10^{-3}\ \text{m} \qquad 39.8\ \text{mm}$

PROBLEM 3.11

3.11 Knowing that each portion of the shaft AD consists of a solid circular rod, determine (a) the portion of the shaft in which the maximum shearing stress occurs, (b) the magnitude of that stress.



SOLUTION

Shaft AB: $T = 400$ lb·in

$c = \frac{1}{2}d = 0.30$ in

$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$\tau_{max} = \frac{(2)(400)}{\pi(0.30)^3} = 9431$ psi

Shaft BC: $T = -400 + 1200 = 800$ lb·in

$c = \frac{1}{2}d = 0.375$ in $\quad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(800)}{\pi(0.375)^3} = 9658$ psi

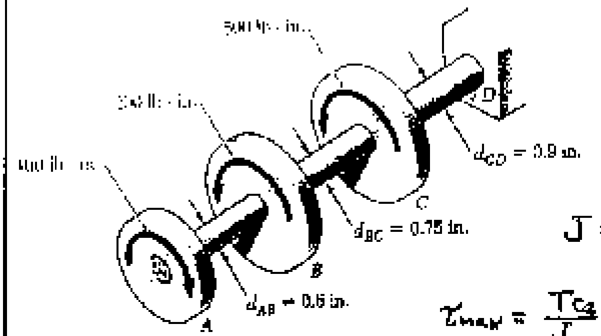
Shaft CD: $T = -400 + 1200 + 500 = 1300$ lb·in

$c = \frac{1}{2}d = 0.45$ in $\quad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1300)}{\pi(0.45)^3} = 9082$ psi

Answers: (a) shaft BC (b) 9.66 ksi ◄

PROBLEM 3.12

3.12 Knowing that a 0.30-in.-diameter hole has been drilled through each portion of shaft AD, determine (a) the portion of the shaft in which the maximum shearing stress occurs, (b) the magnitude of that stress.



SOLUTION

Hole: $c_1 = \frac{1}{2}d_1 = 0.15$ in

Shaft AB: $T = 400$ lb·in

$c_2 = \frac{1}{2}d_2 = 0.30$ in

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.30^4 - 0.15^4)$

$= 0.011928$ in^4

$\tau_{max} = \frac{Tc_2}{J} = \frac{(400)(0.30)}{0.011928} = 10600$ psi ◄

Shaft BC: $T = -400 + 1200 = 800$ lb·in $\quad c_2 = \frac{1}{2}d_2 = 0.375$ in

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.375^4 - 0.15^4) = 0.030268$ in^4

$\tau_{max} = \frac{Tc_2}{J} = \frac{(800)(0.375)}{0.030268} = 9911$ psi

Shaft CD: $T = -400 + 1200 + 500 = 1300$ lb·in $\quad c_2 = \frac{1}{2}d_2 = 0.45$ in

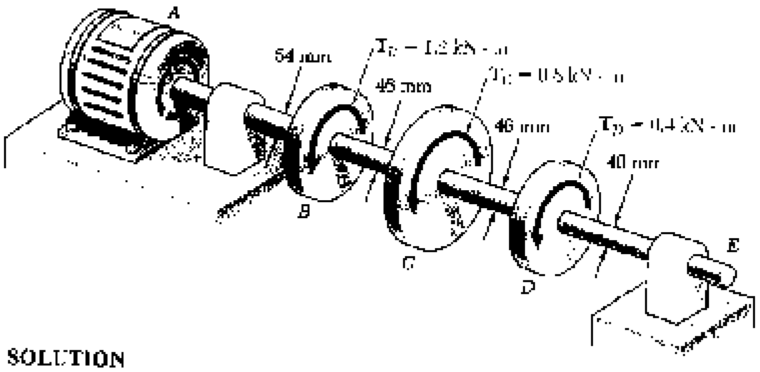
$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.45^4 - 0.15^4) = 0.063617$ in^4

$\tau_{max} = \frac{Tc_2}{J} = \frac{(1300)(0.45)}{0.063617} = 9196$ psi

Answers: (a) shaft AB (b) 10.06 ksi ◄

PROBLEM 3.13

3.13 Under normal operating conditions, the electric motor exerts a torque of 2.4 kN·m at A. Knowing that each shaft is solid, determine the maximum shearing stress (*a*) in shaft AB, (*b*) in shaft BC, (*c*) in shaft CD.



SOLUTION

Shaft AB: $T_{AB} = 2.4\times10^3$ N·m, $c = \frac{1}{2}d = 0.027$ m

$$\tau_{AB} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2.4\times10^3)}{\pi(0.027)^3} = 77.625\times10^6 \text{ Pa}$$ 77.6 MPa ◄

Shaft BC: $T_{BC} = 2.4$ kN·m $- 1.2$ kN·m $= 1.2$ kN·m, $c = \frac{1}{2}d = 0.023$ m

$$\tau_{BC} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1.2\times10^3)}{\pi(0.023)^3} = 62.788\times10^6 \text{ Pa}$$ 62.8 MPa ◄

Shaft CD: $T_{CD} = 0.4\times10^3$ N·m $c = \frac{1}{2}d = 0.023$ m

$$\tau_{CD} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(0.4\times10^3)}{\pi(0.023)^3} = 20.929\times10^6 \text{ Pa}$$ 20.9 MPa ◄

PROBLEM 3.14

3.14 Under normal operating conditions, the electric motor exerts a torque of 2.4 kN·m at A. In order to reduce the mass of the assembly, determine the smallest diameter of shaft BC for which the largest shearing stress in the assembly is not increased.

SOLUTION

See solution to problem 3.14 for figure and for maximum shearing stresses in portions AB, BC, and CD of the shaft. The largest value is $\tau_{max} = 77.625\times10^6$ Pa occuring in AB.

Adjust diameter of BC to obtain the same value of stress

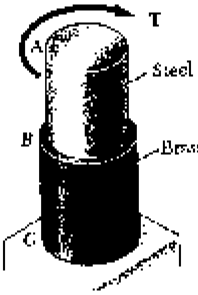
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(1.2\times10^3)}{\pi(77.625\times10^6)} = 9.8415\times10^{-6} \text{ m}^3$$

$c = 21.43\times10^{-3}$ m $\quad d = 2c = 42.8\times10^{-3}$ m 42.8 mm ◄

PROBLEM 3.15

3.15 The allowable stress is 15 ksi in the 1.5-in.-diameter rod AB and 8 ksi in the 1.8-in.-diameter rod BC. Neglecting the effect of stress concentrations, determine the largest torque that may be applied at A.



SOLUTION

$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2} c^4, \quad T = \frac{\pi}{2} c^3 \tau_{max}$

Shaft AB: $\tau_{max} = 15$ ksi $\quad c = \frac{1}{2} d = 0.75$ in

$T = \frac{\pi}{2}(0.75)^3(15) = 9.94$ kip-in

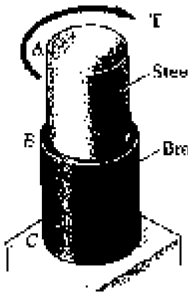
Shaft BC: $\tau_{max} = 8$ ksi $\quad c = \frac{1}{2} d = 0.90$ in

$T = \frac{\pi}{2}(0.90)^3(8) = 9.16$ kip-in.

The allowable torque is the smaller value $\quad T = 9.16$ kip-in ◄

PROBLEM 3.16

3.16 The allowable stress is 15 ksi in the steel rod AB and 8 ksi in the brass rod BC. Knowing that a torque $T = 10$ kip·in. is applied at A, determine the required diameter of (*a*) rod AB, (*b*) for BC.



SOLUTION

$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}, \quad c^3 = \frac{2T}{\pi \tau_{max}}$

Shaft AB: $T = 10$ kip·in $\quad \tau_{max} = 15$ ksi

$c^3 = \frac{(2)(10)}{\pi(15)} = 0.4244$ in^3

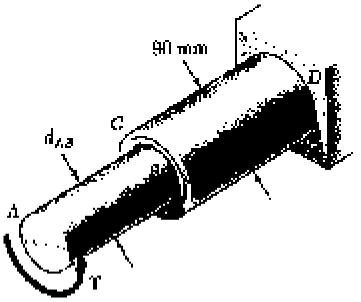
$c = 0.7515$ in $\quad d = 2c = 1.503$ in. ◄

Shaft BC: $T = 10$ kip·in $\quad \tau_{max} = 8$ ksi

$c^3 = \frac{(2)(10)}{\pi(8)} = 0.79577$ in

$c = 0.9267$ in $\quad d = 2c = 1.853$ in. ◄

PROBLEM 3.17



3.17 The solid rod AB has a diameter d_{AB} = 60 mm. The pipe CD has an outer diameter of 90 mm and a wall thickness of 6 mm. Knowing that both the rod and the pipe are made of a steel for which the allowable shearing stress is 75 MPa, determine the largest torque **T** which may be applied at A.

SOLUTION

$\tau_{all} = 75 \times 10^6$ Pa $\qquad T_{all} = \dfrac{J\tau_{all}}{c}$

Rod AB: $c = \frac{1}{2}d = 0.030$ m, $\quad J = \frac{\pi}{2}c^4$

$T_{all} = \frac{\pi}{2}c^3\tau_{all} = \frac{\pi}{2}(0.030)^3(75\times10^6)$

$= 3.181 \times 10^3$ N·m

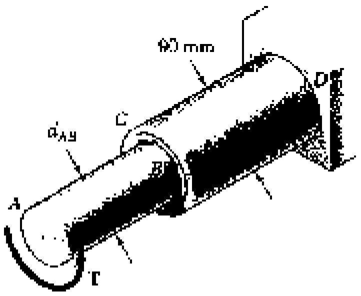
Pipe CD: $c_2 = \frac{1}{2}d_2 = 0.045$ m $\qquad c_1 = c_2 - t = 0.045 - 0.006 = 0.039$ m

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073 \times 10^{-6}$ m^4

$T_{all} = \dfrac{(2.8073\times10^{-6})(75\times10^6)}{0.045} = 4.679 \times 10^3$ N·m

Allowable torque is the smaller value $(3.18 \times 10^3$ N·m$)$ 3.18 kN·m

PROBLEM 3.18



3.18 The solid rod AB has a diameter d_1 = 60 mm and is made of a steel for which the allowable shearing stress is 85 MPa. The pipe CD has an outer diameter of 90 mm and a wall thickness of 6 mm; it is made of an aluminum for which the allowable shearing stress is 54 MPa. Determine the largest torque **T** which may be applied at A.

SOLUTION

Rod AB: $\tau_{all} = 85 \times 10^6$ Pa, $\quad c = \frac{1}{2}d = 0.030$ m

$T_{all} = \dfrac{J\tau_{all}}{c} = \frac{\pi}{2}c^3\tau_{all}$

$= \frac{\pi}{2}(0.030)^3(85\times10^6) = 3.605 \times 10^3$ N·m

Pipe CD: $\tau_{all} = 54 \times 10^6$ Pa $\qquad c_2 = \frac{1}{2}d_2 = 0.045$ m

$c_1 = c_2 - t = 0.045 - 0.006 = 0.039$ m

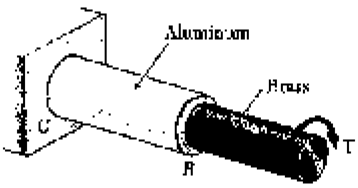
$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073 \times 10^{-6}$ m^4

$T_{all} = \dfrac{J\tau_{all}}{c_2} = \dfrac{(2.8073\times10^{-6})(54\times10^6)}{0.045} = 3.369 \times 10^3$ N·m

Allowable torque is smaller value $T_{all} = 3.369 \times 10^3$ N·m

3.37 kN·m

PROBLEM 3.19



3.19 The allowable stress is 50 MPa in the brass rod AB and 25 MPa in the aluminum rod BC. Knowing that a torque $T = 1250$ N·m is applied at A, determine the required diameter of (*a*) rod AB, (*b*) for BC.

SOLUTION

$\tau_{max} = \frac{Tc}{J}$ $\quad J = \frac{\pi}{2}c^4$ $\quad c^3 = \frac{2T}{\pi \tau_{max}}$

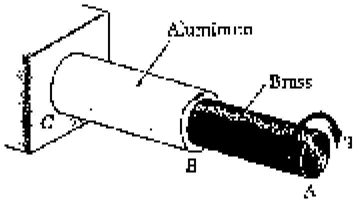
Rod AB: $c^3 = \frac{(2)(1250)}{\pi(50\times10^6)} = 15.915\times10^{-6}\ m^3$

$c = 25.15\times10^{-3}\ m = 25.15\ mm$ $\quad d_{AB} = 2c = 50.3\ mm$ ◄

Rod BC: $c^3 = \frac{(2)(1250)}{\pi(25\times10^6)} = 31.831\times10^{-6}\ m^3$

$c = 31.69\times10^{-3}\ m = 31.69\ mm$ $\quad d_{BC} = 2c = 63.4\ mm$ ◄

PROBLEM 3.20



3.20 The *solid* rod BC has a diameter of 30 mm and is made an aluminum for which the allowable shearing stress is 25 MPa. Rod AB is *hollow* and has an outer diameter of 25 mm; it is made a brass for which the allowable shearing stress is 50 MPa. Determine (*a*) the largest inner diameter of rod AB for which the factor of safety is the same for each rod, (*b*) the largest torque that may be applied at A.

SOLUTION

Solid rod BC: $\tau = \frac{Tc}{J}$ $\quad J = \frac{\pi}{2}c^4$

$\tau_{all} = 25\times10^6\ Pa$ $\quad c = \frac{1}{2}d = 0.015\ m$

$T_{all} = \frac{\pi}{2}c^3\tau_{all} = \frac{\pi}{2}(0.015)^3(25\times10^6) = 132.536\ N\cdot m$

Hollow rod AB: $\tau_{all} = 50\times10^6\ Pa$ $\quad T_{all} = 132.536\ N\cdot m$

$c_2 = \frac{1}{2}d_2 = \frac{1}{2}(0.025) = 0.0125\ m$ $\quad c_1 = ?$

$T_{all} = \frac{J\,\tau_{all}}{c_2} = \frac{\pi}{2}(c_2^4 - c_1^4)\frac{\tau_{all}}{c_2}$

$c_1^4 = c_2^4 - \frac{2T_{all}c_2}{\pi\tau_{all}} = 0.0125^4 - \frac{(2)(132.536)(0.0125)}{\pi(50\times10^6)} = 3.3203\times10^{-9}\ m^4$

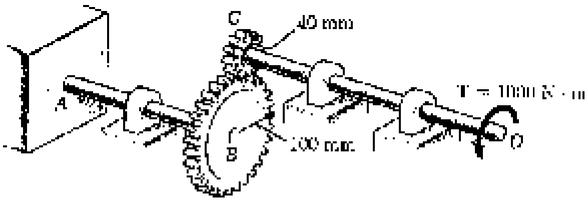
$c_1 = 7.59\times10^{-3}\ m = 7.59\ mm$ $\quad d_1 = 2c_1 = 15.18\ mm$ ◄

Allowable torque $\quad T_{all} = 132.5\ N\cdot m$ ◄

PROBLEM 3.21

3.21 A torque of magnitude $T = 1000$ N·m is applied at D as shown. Knowing that the diameter of shaft AB is 56 mm and the diameter of shaft CD is 42 mm, determine the maximum shearing stress in (*a*) shaft AB, (*b*) shaft CD.

SOLUTION



$T_{CD} = 1000$ N·m

$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500$ N·m

Shaft AB: $c = \frac{1}{2}d = 0.028$ m

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2500)}{\pi(0.028)^3} = 72.50 \times 10^6$ 72.5 MPa ◄

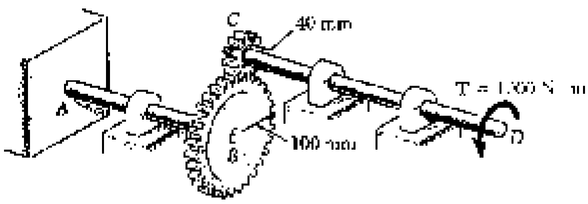
Shaft BC: $c = \frac{1}{2}d = 0.020$ m

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1000)}{\pi(0.020)^3} = 68.7 \times 10^6$ 68.7 MPa ◄

PROBLEM 3.22

3.22 A torque of magnitude $T = 1000$ N·m is applied at D as shown. Knowing that the allowable shearing stress is 60 MPa in each shaft, determine the required diameter of (*a*) shaft AB, (*b*) shaft CD.

SOLUTION



$T_{CD} = 1000$ N·m

$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500$ N·m

Shaft AB: $\tau_{all} = 60 \times 10^6$ Pa

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$ $c^3 = \frac{2T}{\pi \tau} = \frac{(2)(2500)}{\pi(60 \times 10^6)} = 26.526 \times 10^{-6}$ m^3

$c = 29.82 \times 10^{-3} = 29.82$ mm $d = 2c = 59.6$ mm ◄

Shaft CD: $\tau_{all} = 60 \times 10^6$ Pa

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$ $c^3 = \frac{2T}{\pi \tau} = \frac{(2)(1000)}{\pi(60 \times 10^6)} = 10.610 \times 10^{-6}$ m^3

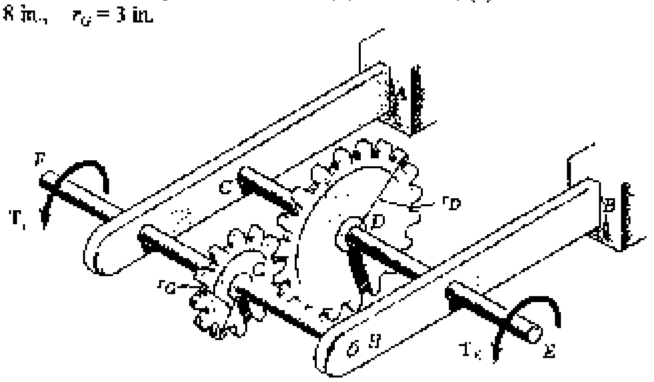
$c = 21.97 \times 10^{-3}$ m $= 21.97$ mm $d = 2c = 43.9$ mm ◄

PROBLEM 3.23

3.23 and 3.24 Under normal operating conditions a motor exerts a torque of magnitude $T_F = 1200$ lb·in. at F. Knowing that the allowable shearing stress is 10.5 ksi in each shaft, for the given data, determine the required diameter of (*a*) shaft *CDE*, (*b*) shaft *FGH*.

3.23 $r_D = 8$ in., $r_G = 3$ in.

SOLUTION



$T_F = 1200$ lb·in

$T_E = \frac{r_D}{r_G} T_F = \frac{8}{3}(1200) = 3200$ lb·in

$\tau_{all} = 10.5$ ksi $= 10500$ psi

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$, $\quad c^3 = \frac{2T}{\pi \tau}$

(a) Shaft CDE

$c^3 = \frac{(2)(3200)}{\pi(10500)} = 0.194012 \text{ in}^3$

$c = 0.5789 \qquad d_{DE} = 2c = 1.158$ in. ◄

(b) Shaft FGH

$c^3 = \frac{(2)(1200)}{\pi(10500)} = 0.072757 \text{ in}^3$

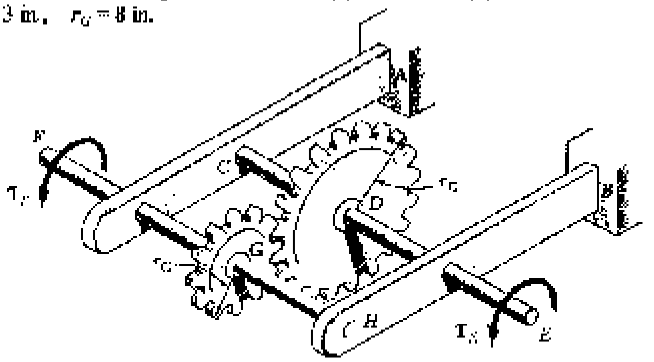
$c = 0.4174$ in $\qquad d_{FG} = 2c = 0.835$ in ◄

PROBLEM 3.24

3.23 and 3.24 Under normal operating conditions a motor exerts a torque of magnitude $T_F = 1200$ lb·in. at F. Knowing that the allowable shearing stress is 10.5 ksi in each shaft, for the given data, determine the required diameter of (*a*) shaft *CDE*, (*b*) shaft *FGH*.

3.24 $r_D = 3$ in., $r_G = 8$ in.

SOLUTION



$T_F = 1200$ lb·in

$T_E = \frac{r_D}{r_G} T_F = \frac{3}{8}(1200) = 450$ lb·in.

$\tau_{all} = 10.5$ ksi $= 10500$ psi

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi \tau}$

Shaft CDE

$c^3 = \frac{(2)(450)}{\pi(10500)} = 0.027284 \text{ in}^3$

$c = 0.30105$ in $\qquad d_{DE} = 2c = 0.602$ in ◄

Shaft FGH

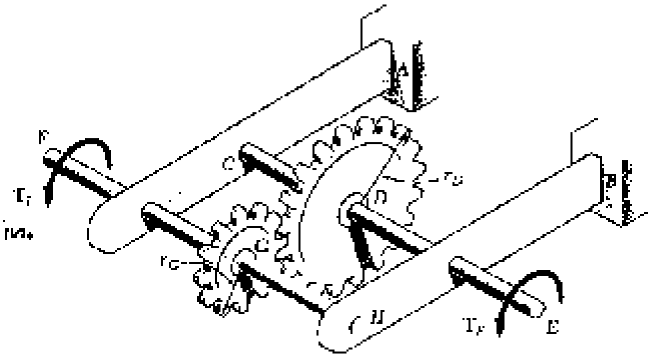
$c^3 = \frac{(2)(1200)}{\pi(10500)} = 0.072757 \text{ in}^3$

$c = 0.4174$ in. $\qquad d_{FG} = 2c = 0.835$ in ◄

PROBLEM 3.25

3.25 Under normal operating conditions a motor exerts a torque of magnitude T_F at F. The shafts are made of a steel for which the allowable shearing stress is 12 ksi and have diameters of $d_{CDE} = 0.900$ in. and $d_{FGH} = 0.800$ in. Knowing that $r_D = 6.5$ in. and $r_G = 4.5$ in., determine the largest torque T_F which may be exerted at F.

SOLUTION



$\tau_{all} = 12$ ksi

Shaft FG: $c = \frac{1}{2}d = 0.400$ in

$T_{F,all} = \frac{J\tau_{all}}{c} = \frac{\pi}{2}c^3\tau_{all}$

$= \frac{\pi}{2}(0.400)^3(12) = 1.206$ Kip in.

Shaft DE: $c = \frac{1}{2}d = 0.450$ in

$T_{E,all} = \frac{\pi}{2}c^3\tau_{all}$

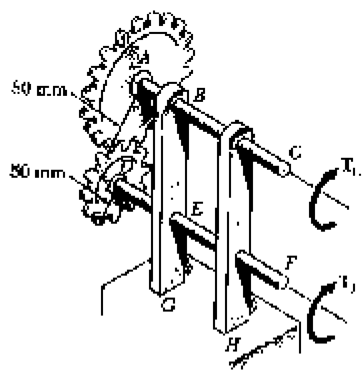
$= \frac{\pi}{2}(0.450)^3(12) = 1.7177$ kip-in

$T_F = \frac{r_G}{r_D}T_E \qquad T_{F,all} = \frac{4.5}{6.5}(1.7177) = 1.189$ kip-in

Allowable value of T_F is the smaller $\qquad T_{F,all} = 1.189$ Kip-in

PROBLEM 3.26

3.26 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 60 MPa. Knowing that a 600 N·m torque T_C is applied at C, determine the required diameter of (*a*) shaft BC, (*b*) shaft EF.



SOLUTION

Shaft BC: $T_C = 600$ N·m, $\tau_{max} = 60\times10^6$ Pa

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(600)}{\pi(60\times10^6)} = 6.3662\times10^{-6}\ m^3$

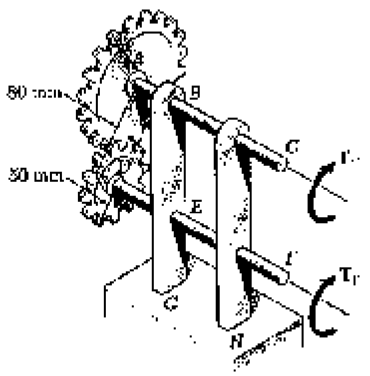
$c = 18.53\times10^{-3}$ m $= 18.53$ mm, $d_{BC} = 2c = 37.1$ mm

Shaft EF: $T_F = \frac{r_E}{r_A}T_C = \frac{50}{80}(600) = 375$ N·m

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(375)}{\pi(60\times10^6)} = 3.9787\times10^{-6}\ m^3$

$c = 15.85\times10^{-3}$ m $= 15.85$ mm, $d_{EF} = 2c = 31.7$ mm

PROBLEM 3.27



3.27 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 50 MPa. Knowing that the diameters of the two shafts are, respectively, $d_{BC} = 40$ mm and $d_{EF} = 32$ mm, determine the largest torque T_C which may be applied at C.

SOLUTION

Shaft AC: $\tau_{max} = 50\times10^6$ Pa, $c = \frac{1}{2}d = 0.020$ m

$$T_C = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.020)^3(50\times10^6)$$

$$= 628.3 \text{ N-m}$$

Shaft DF: $\tau_{max} = 50\times10^6$ Pa, $c = \frac{1}{2}d = 0.016$ m

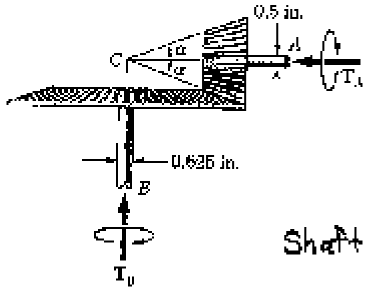
$$T_F = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.016)^3(50\times10^6)$$

$$= 321.7 \text{ N-m}$$

From Statics: $T_C = \frac{r_A}{r_D}T_F = \frac{80}{50}(321.7) = 514.7$ N-m

Allowable value of T_C is the smaller, i.e. $T_F = 515$ N-m

PROBLEM 3.28



3.28 In the bevel-gear system shown $\alpha = 18.43°$. Knowing that the allowable shearing stress is 8 ksi in each shaft, determine the largest torque T_A which may be applied at A.

SOLUTION

Shaft A: $\tau = 8$ ksi $\quad c = \frac{1}{2}d = 0.25$ in

$$T_A = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.25)^3(8) = 0.19635 \text{ kip-in}$$

Shaft B: $\tau = 8$ ksi $\quad c = \frac{1}{2}d = 0.3125$ in

$$T_B = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.3125)^3(8) = 0.3835 \text{ kip-in}$$

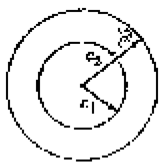
From Statics: $T_A = \frac{r_A}{r_B}T_B = (\tan\alpha)T_B = (\tan 18.43°)(0.3885)$

$$= 0.12779 \text{ kip-in}$$

Allowable value of T_A is the smaller

$$T_A = 0.1278 \text{ kip-in} = 127.8 \text{ lb-in}$$

PROBLEM 3.29



3.29 (*a*) For a given allowable stress, determine the ratio T/w of the maximum allowable torque T and the weight per unit length w for the hollow shaft shown. (*b*) Denoting by $(T/w)_0$ the value of this ratio computed for a solid shaft of the same radius c_2, express the ratio T/w for the hollow shaft in terms of $(T/w)_0$ and c_1/c_2.

SOLUTION

w = weight per unit length, γ = specific weight

W = total weight, L = length

$$w = \frac{W}{L} = \frac{\gamma L A}{L} = \gamma A = \gamma \pi (c_2^2 - c_1^2)$$

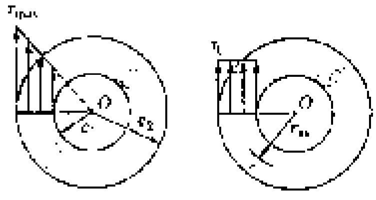
$$T = \frac{J \tau_{al}}{c_2} = \frac{\pi}{2} \frac{c_2^4 - c_1^4}{c_2} \tau_{al} = \frac{\pi}{2} \frac{(c_2^2+c_1^2)(c_2^2-c_1^2)}{c_2} \tau_{al}$$

$$\left(\frac{T}{w}\right)_h = \frac{(c_2^2+c_1^2)\tau_{al}}{2\gamma c_2} = \frac{c_2 \tau_{al}}{2\gamma}\left(1 + \frac{c_1^2}{c_2^2}\right) \quad \text{(hollow shaft)}$$

$c_1 = 0$ for solid shaft $\quad \left(\frac{T}{w}\right)_0 = \frac{c_2 \tau_{al}}{2\gamma} \quad$ (solid shaft)

$$\frac{(T/w)_h}{(T/w)_0} = 1 + \frac{c_1^2}{c_2^2} \qquad \left(\frac{T}{w}\right)_h = \left(\frac{T}{w}\right)_0 \left(1 + \frac{c_1^2}{c_2^2}\right)$$

PROBLEM 3.30



(*a*) (*b*)

3.30 While the exact distribution of the shearing stresses in a hollow cylinder shaft is a shown in Fig. (1), an approximate value may be obtained for τ_{max} by assuming the stresses to be uniformly distributed over the area A of the cross section, as shown in Fig. (2), and then further assuming that all the elementary shearing forces act a distance from O equal to the mean radius $r_m = \frac{1}{2}(c_1 + c_2)$ of the cross section. This approximate value is $\tau_0 = T/Ar_m$, where T is the applied torque. Determine the ratio τ_{max}/τ_0 of the true value of the maximum shearing stress and its approximate value τ_0 for values of c_1/c_2 respectively equal to 1.00, 0.95, 0.75, 0.50, and 0.

SOLUTION

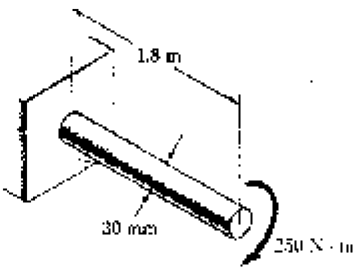
For a hollow shaft: $$\tau_{max} = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)} = \frac{2Tc_2}{\pi(c_2^2-c_1^2)(c_2^2+c_1^2)} = \frac{2Tc_2}{A(c_2^2+c_1^2)}$$

By definition $$\tau_0 = \frac{T}{A r_m} = \frac{2T}{A(c_2+c_1)}$$

Dividing $$\frac{\tau_{max}}{\tau_0} = \frac{c_2(c_2+c_1)}{c_2^2+c_1^2} = \frac{1+(c_1/c_2)}{1+(c_1/c_2)^2}$$

c_1/c_2	1.0	0.95	0.75	0.5	0.0
τ_{max}/τ_0	1.0	1.025	1.120	1.200	1.0

PROBLEM 3.31



3.31 (*a*) For the solid steel shaft shown (G = 77 GPa), determine the angle of twist at A. (*b*) Solve part *a*, assuming that the steel shaft is hollow with a 30-mm outer diameter and a 20-mm inner diameter.

SOLUTION

(a) $c = \frac{1}{2}d = 0.015 \text{ m}, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4$

$J = 79.522 \times 10^{-9} \text{ m}^4, \quad L = 1.8 \text{ m}, \quad G = 77 \times 10^9 \text{ Pa}$

$T = 250 \text{ N}\cdot\text{m} \qquad \varphi = \frac{TL}{GJ}$

$$\varphi = \frac{(250)(1.8)}{(77\times10^9)(79.522\times10^{-9})} = 73.49\times10^{-3} \text{ rad}$$

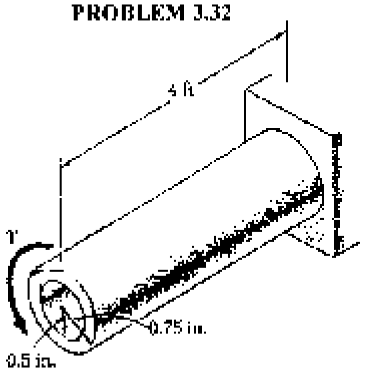
$$\varphi = \frac{(73.49\times10^{-3})180}{\pi} = 4.21°$$

(b) $c_2 = 0.015 \text{ m}, \quad c_1 = \frac{1}{2}d_1 = 0.010 \text{ m}, \quad J = \frac{\pi}{2}(c_2^4 - c_1^4)$

$J = \frac{\pi}{2}(0.015^4 - 0.010^4) = 63.814\times10^{-9} \text{ m}^4 \qquad \varphi = \frac{TL}{GJ}$

$$\varphi = \frac{(250)(1.8)}{(77\times10^9)(63.814\times10^{-9})} = 91.58\times10^{-3} \text{ rad} = \frac{180}{\pi}(91.58\times10^{-3}) = 5.25°$$

PROBLEM 3.32



3.32 For the aluminum shaft shown ($G = 3.9 \times 10^6$ psi), determine (*a*) the torque **T** which causes an angle of twist of 5°, (*b*) the angle of twist caused by the same torque **T** in a solid cylindrical shaft of the same length and cross-sectional area.

SOLUTION

(a) $\varphi = \frac{TL}{GJ}, \qquad T = \frac{GJ\varphi}{L}$

$\varphi = 5° = 87.266\times10^{-3} \text{ rad}, \qquad L = 4 \text{ ft} = 48 \text{ in}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.75^4 - 0.5^4) = 0.39884 \text{ in}^4$

$$T = \frac{(3.9\times10^6)(0.39884)(87.266\times10^{-3})}{48}$$

$= 2.8279\times10^3 \text{ lb}\cdot\text{in} = 2.83 \text{ kip}\cdot\text{in}$

(b) Hollow shaft $A = \pi(c_2^2 - c_1^2)$ Solid shaft $A = \pi c^2$

Matching areas $c^2 = c_2^2 - c_1^2 = 0.75^2 - 0.5^2 = 0.3125 \text{ in}^2$

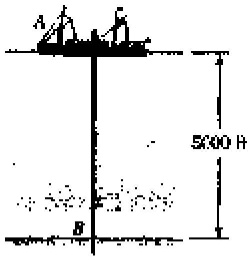
$c = 0.5590 \text{ in}, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.5590)^4 = 153.398\times10^{-3} \text{ in}^4$

$$\varphi = \frac{TL}{GJ} = \frac{(2.8279\times10^3)(48)}{(3.9\times10^6)(153.398\times10^{-3})} = 226.89\times10^{-3} \text{ rad}$$

$= 13.00°$

PROBLEM 3.33

3.33 The ship at A has just started to drill for oil on the ocean floor at a depth of 5000 ft. Knowing that the top of the 8-in.-diameter steel drill pipe ($G = 11.2 \times 10^6$ psi) rotates through two complete revolutions before the drill bit at B starts to operate, determine the maximum shearing stress caused in the pipe by torsion.



SOLUTION

$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L}$$

$$\tau = \frac{Tc}{J} = \frac{GJ\varphi c}{JL} = \frac{G\varphi c}{L}$$

$\varphi = 2$ rev $= (2)(2\pi) = 12.566$ rad, $\quad c = \frac{1}{2}d = 4.0$ in

$L = 5000$ ft $= 60000$ in $\quad \tau = \dfrac{(11.2\times10^6)(12.566)(4.0)}{60000}$

$= 9.3826\times10^3$ psi $=$ 9.38 ksi

PROBLEM 3.34

3.34 Determine the largest allowable diameter of a 3-m-long steel rod ($G = 77$ GPa) if the rod is to be twisted through 30° without exceeding a shearing stress of 80 MPa.

SOLUTION

$L = 3$ m, $\quad \varphi = \dfrac{30\pi}{180} = 523.6\times10^{-3}$ rad, $\quad \tau = 80\times10^6$ Pa

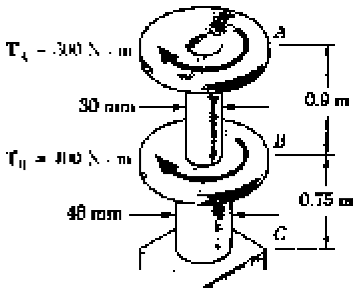
$\varphi = \dfrac{TL}{GJ}$, $\quad T = \dfrac{GJ\varphi}{L}$, $\quad \tau = \dfrac{Tc}{J} = \dfrac{GJ\varphi c}{JL} = \dfrac{G\varphi c}{L}$; $\quad c = \dfrac{\tau L}{G\varphi}$

$c = \dfrac{(80\times10^6)(3.0)}{(77\times10^9)(523.6\times10^{-3})} = 5.953\times10^{-3}$ m $= 5.953$ mm

$d = 2c = 11.91$ mm

PROBLEM 3.35

3.35 The torques shown are exerted on pulleys A and B. Knowing that the shafts are solid and made of aluminum ($G = 77$ GPa), determine the angle of twist between (*a*) A and B, (*b*) A and C.



SOLUTION

(a) $T_{AB} = 300$ N·m, $L_{AB} = 0.9$ m, $c_{AB} = \frac{1}{2}d = 0.015$ m

$J_{AB} = \frac{\pi}{2}(0.015)^4 = 79.522\times10^{-9}$ m^4

$\varphi_{AB} = \dfrac{T_{AB}L_{AB}}{GJ} = \dfrac{(300)(0.9)}{(77\times10^9)(79.522\times10^{-9})} = 44.095\times10^{-3}$ rad

$\varphi_{AB} = 2.53°$

(b) $T_{BC} = 300 + 400 = 700$ N·m, $L_{BC} = 0.75$ m, $c_{BC} = \frac{1}{2}d = 0.023$ m

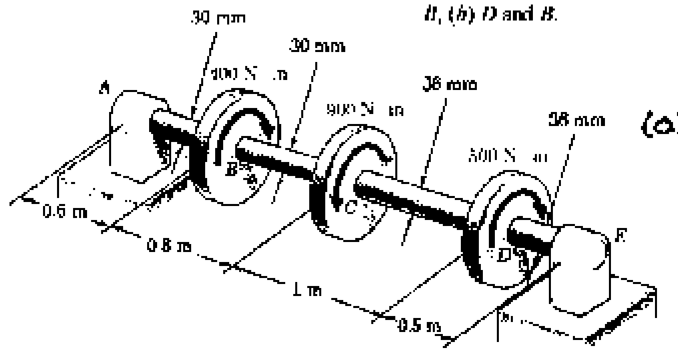
$J_{BC} = \frac{\pi}{2}(0.023)^4 = 439.573\times10^{-9}$ m^4

$\varphi_{BC} = \dfrac{T_{BC}L_{BC}}{GJ_{BC}} = \dfrac{(700)(0.75)}{(77\times10^9)(439.573\times10^{-9})} = 15.511\times10^{-3}$ rad

$\varphi_{AC} = \varphi_{AB} + \varphi_{BC} = 59.606\times10^{-3}$ rad $= 3.42°$

PROBLEM 3.36

3.36 The torques shown are exerted on pulleys B, C and D. Knowing that the entire shaft is made of steel ($G = 27$ GPa), determine the angle of twist between (a) C and B, (b) D and B.



SOLUTION

(a) Shaft BC: $c = \frac{1}{2}d = 0.015\ m$

$J_{BC} = \frac{\pi}{2}c^4 = 79.522 \times 10^{-9}\ m^4$

$L_{BC} = 0.8\ m, \quad G = 27 \times 10^9\ Pa$

$$\varphi_{BC} = \frac{TL}{GJ} = \frac{(400)(0.8)}{(27 \times 10^9)(79.522 \times 10^{-9})}$$

$= 0.149904\ rad = 8.54°$

(b) Shaft CD: $c = \frac{1}{2}d = 0.018\ m \qquad J_{CD} = \frac{\pi}{2}c^4 = 164.896 \times 10^{-9}\ m^4$

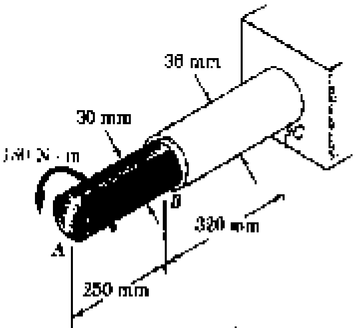
$L_{CD} = 1.0\ m \qquad T_{CD} = 400 - 900 = -500\ N \cdot m$

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(-500)(1.0)}{(27 \times 10^9)(164.896 \times 10^{-9})} = -0.11230\ rad$$

$\varphi_{BD} = \varphi_{BC} + \varphi_{CD} = 0.14904 - 0.11230 = 0.03674\ rad = 2.11°$

PROBLEM 3.37

3.37 The solid brass rod AB ($G = 39$ GPa) is bonded to the solid aluminum rod BC ($G = 27$ GPa). Determine the angle of twist (a) at B, (b) at A.



SOLUTION

Shaft AB: $c = \frac{1}{2}d = 0.015\ m \qquad L = 0.250\ m$

$G = 39 \times 10^9\ Pa \qquad T = 180\ N \cdot m$

$J = \frac{\pi}{2}c^4 = 79.522 \times 10^{-9}\ m^4$

$$\varphi_{AB} = \frac{TL}{GJ} = \frac{(180)(0.250)}{(39 \times 10^9)(79.522 \times 10^{-9})} = 14.510 \times 10^{-3}\ rad$$

Shaft BC: $c = \frac{1}{2}d = 0.018\ m, \quad L = 0.320\ m$

$G = 27 \times 10^9\ Pa, \quad T = 180\ N \cdot m$

$J = \frac{\pi}{2}c^4 = 164.896 \times 10^{-9}\ m^4$

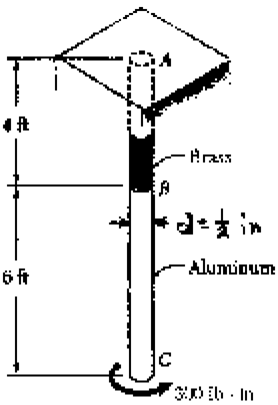
$$\varphi_{BC} = \frac{(180)(0.320)}{(27 \times 10^9)(164.896 \times 10^{-9})} = 12.937 \times 10^{-3}\ rad$$

Answers: (a) $\varphi_B = \varphi_{BC} = 12.937 \times 10^{-3}\ rad = 0.741°$

(b) $\varphi_A = \varphi_{BC} + \varphi_{AB} = 27.447 \times 10^{-3}\ rad = 1.573°$

PROBLEM 3.38

3.38 The brass rod AB ($G = 5.6 \times 10^6$ psi) is bonded to the aluminum rod BC ($G = 3.9 \times 10^6$ psi). Knowing that each rod is solid, determine the angle of twist (a) at B, (b) at C.



SOLUTION

Both portions $c = \frac{1}{2}d = 0.25$ in

$J = \frac{\pi}{2}c^4 = 6.1359 \times 10^{-3}$ in^4 $\qquad T = 300$ lb·in

Shaft AB: $G_{AB} = 5.6 \times 10^6$ psi $\quad L_{AB} = 4$ ft $= 48$ in

$$\varphi_B = \varphi_{AB} = \frac{T L_{AB}}{G_{AB} J} = \frac{(300)(48)}{(5.6\times10^6)(6.1359\times10^{-3})}$$

$$= 0.419 \text{ rad} = 24.0°$$

Shaft BC: $G = 3.9 \times 10^6$ psi $\quad L_{BC} = 6$ ft $= 72$ in.

$$\varphi_{BC} = \frac{T L_{BC}}{G_{BC} J} = \frac{(300)(72)}{(3.9\times10^6)(6.1359\times10^{-3})}$$

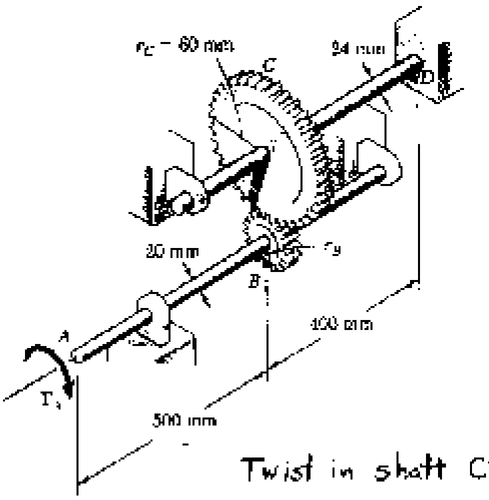
$$= 0.903 \text{ rad} = 51.7°$$

$$\varphi_C = \varphi_B + \varphi_{BC}$$

$$= 0.417 + 0.903 = 1.320 \text{ rad} = 75.6°$$

PROBLEM 3.39

3.39 Two solid steel shafts ($G = 77$ GPa) are connected by the gears shown. Knowing that the radius of gear B is $r_B = 20$ mm, determine the angle through which end A rotates when $T_A = 75$ N·m.



SOLUTION

Calculation of torques

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \quad \therefore \quad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = T_A = 75 \text{ N·m}$$

$$T_{CD} = \frac{0.060}{0.020}(75) = 225 \text{ N·m}$$

Twist in shaft CD

$$J_{CD} = \frac{\pi}{2} c_{CD}^4 = \frac{\pi}{2}(0.012)^4 = 32.572 \times 10^{-9} \text{ m}^4, \qquad L_{CD} = 0.400 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa} \qquad \varphi_{CD} = \frac{TL}{GJ} = \frac{(225)(0.400)}{(77\times10^9)(32.572\times10^{-9})} = 35.885 \times 10^{-3} \text{ rad}$$

Rotation angle at C $\qquad \varphi_C = \varphi_{CD} = 35.885 \times 10^{-3}$ rad

Circumferential displacement at contact points of gears B and C

$$\delta = r_C \varphi_C = r_B \varphi_B$$

Rotation angle at B: $\quad \varphi_B = \frac{r_C}{r_B}\varphi_C = \frac{0.060}{0.020}(35.885\times10^{-3}) = 107.654\times10^{-3}$ rad

Twist in shaft AB:

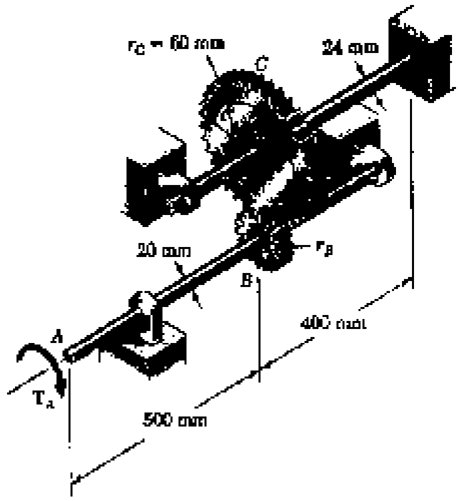
$$J_{AB} = \frac{\pi}{2} c_{AB}^4 = \frac{\pi}{2}(0.010)^4 = 15.708 \times 10^{-9} \text{ m}^4, \qquad L_{AB} = 0.500 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa}, \quad \varphi_{AB} = \frac{TL}{GJ} = \frac{(75)(0.500)}{(77\times10^9)(15.708\times10^{-9})} = 31.004\times10^{-3} \text{ rad}$$

Rotation at A $\qquad \varphi_A = \varphi_B + \varphi_{AB} = 138.7 \times 10^{-3} \text{ rad} = 7.94°$

PROBLEM 3.40

3.40 Solve Prob. 3.39, assuming that a change in design of the assembly resulted in the radius of gear *B* being increased to 30 mm.



$G = 77$ GPa, $r_B = 30$ mm, $T_A = 75$ N·m

Determine the angle through which end A rotates.

SOLUTION

Calculation of torques

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \quad \therefore \quad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = T_A = 75 \text{ N·m}$$

$$T_{CD} = \frac{0.060}{0.030}(75) = 150 \text{ N·m}$$

Twist in shaft CD

$$J_{CD} = \frac{\pi}{2} c_{CD}^4 = \frac{\pi}{2}(0.012)^4 = 32.572 \times 10^{-9} \text{ m}^4, \qquad L_{CD} = 0.400 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa}, \qquad \varphi_{CD} = \frac{TL}{GJ} = \frac{(150)(0.400)}{(77 \times 10^9)(32.572 \times 10^{-9})} = 23.923 \times 10^{-3} \text{ rad}$$

Rotation angle at C $\quad \varphi_C = \varphi_{CD} = 23.923 \times 10^{-3}$ rad.

Circumferential displacement at contact points of gears B and C.

$$\delta = r_C \varphi_C = r_B \varphi_B$$

Rotation angle at B $\quad \varphi_B = \frac{r_C}{r_B}\varphi_C = \frac{0.060}{0.030}(23.923 \times 10^{-3}) = 47.846 \times 10^{-3}$ rad

Twist in shaft AB

$$J_{AB} = \frac{\pi}{2} c_{AB}^4 = \frac{\pi}{2}(0.010)^4 = 15.708 \times 10^{-9} \text{ m}^4, \qquad L_{AB} = 0.500 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa}, \qquad \varphi_{AB} = \frac{TL}{GJ} = \frac{(75)(0.500)}{(77 \times 10^9)(15.708 \times 10^{-9})} = 31.004 \times 10^{-3} \text{ rad}$$

Rotation at A $\quad \varphi_A = \varphi_B + \varphi_{AB} = 78.85 \times 10^{-3} \text{ rad} = 4.52°$ ◄