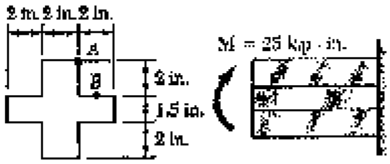
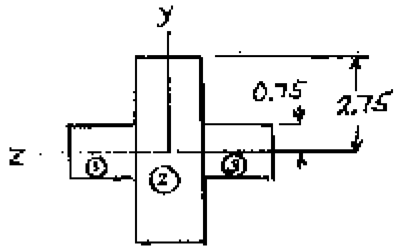
4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

PROBLEM 4.1



SOLUTION



For rectangle $I = \frac{1}{12} b h^3$

For cross sectional area

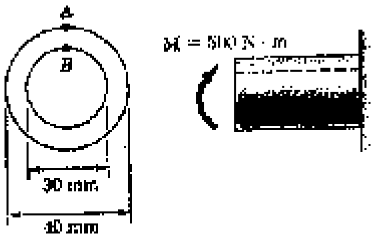
$$I = I_1 + I_2 + I_3 = \frac{1}{12}(2)(1.5)^3 + \frac{1}{12}(2)(5.5)^3 + \frac{1}{12}(2)(1.5)^3 = 28.854 \text{ in}^4$$

(a) $y_A = 2.75 \text{ in}$ $\qquad \sigma_A = -\frac{M y_A}{I} = -\frac{(25)(2.75)}{28.854} = -2.38 \text{ ksi}$

(b) $y_B = 0.75 \text{ in.}$ $\qquad \sigma_B = -\frac{M y_B}{I} = -\frac{(25)(0.75)}{28.854} = -0.650 \text{ ksi}$

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

PROBLEM 4.2



SOLUTION

$r_i = \frac{1}{2} d_i = 15 \text{ mm} \qquad r_o = \frac{1}{2} d_o = 20 \text{ mm}$

$$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(20^4 - 15^4)$$

$$= 85.903 \times 10^3 \text{ mm}^4 = 85.903 \times 10^{-9} \text{ m}^4$$

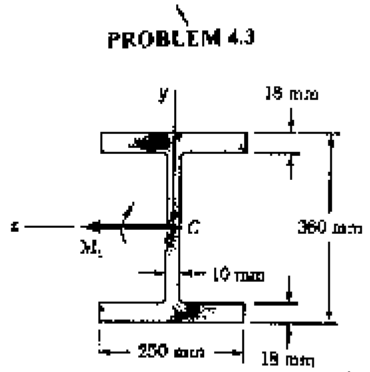
(a) $y_A = 20 \text{ mm} = 0.020 \text{ m}$ $\qquad \sigma_A = -\frac{M y_A}{I} = -\frac{(500)(0.020)}{85.903 \times 10^{-9}}$

$$= -116.4 \times 10^6 \text{ Pa} = -116.4 \text{ MPa}$$

(b) $y_B = 15 \text{ mm} = 0.015 \text{ m}$ $\qquad \sigma_B = -\frac{M y_B}{I} = -\frac{(500)(0.015)}{85.903 \times 10^{-9}}$

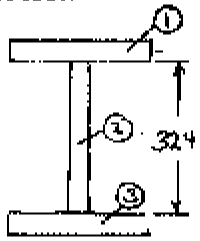
$$= -87.3 \times 10^6 \text{ Pa} = -87.3 \text{ MPa}$$

PROBLEM 4.3



4.3 The wide-flange beam shown is made of a high-strength, low-alloy steel for which σ_Y = 345 MPa and σ_U = 450 MPa. Using a factor of safety of 3.0, determine the largest couple that can be applied to the beam when it is bent about the z axis. Neglect the effect of fillets.

SOLUTION



$$I_1 = \tfrac{1}{12} b h^3 + A d^2 = \tfrac{1}{12}(250)(18^3) + (250)(18)(171)^2 = 131.706 \times 10^6 \text{ mm}^4$$

$$I_2 = \tfrac{1}{12}(10)(324)^3 = 28.344 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 131.706 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 291.76 \times 10^6 \text{ mm}^4 = 291.76 \times 10^{-6} \text{ m}^4$$

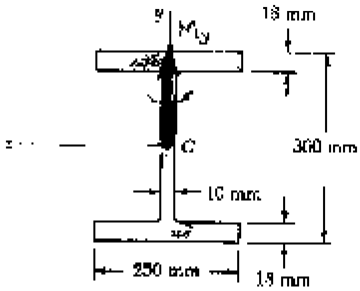
$$\sigma = \frac{Mc}{I} \quad \text{where} \quad c = \frac{360}{2} = 180 \text{ mm} = 0.180 \text{ m}$$

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{450 \times 10^6}{3.0} = 150 \times 10^6 \text{ Pa}$$

$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(150 \times 10^6)(291.76 \times 10^{-6})}{0.180} = 243 \times 10^3 \text{ N}\cdot\text{m}$$

$$= 243 \text{ kN}\cdot\text{m}$$

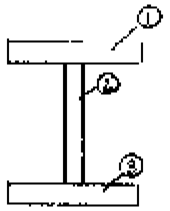
PROBLEM 4.4



4.3 The wide-flange beam shown is made of a high-strength, low-alloy steel for which $\sigma_Y = 345$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.0, determine the largest couple that can be applied to the beam when it is bent about the z axis. Neglect the effect of fillets.

4.4 Solve Prob. 4.3, assuming that is bent about the y axis.

SOLUTION



$$I_1 = \frac{1}{12}(18)(250)^3$$
$$= 23.438 \times 10^6 \quad mm^4$$

$$I_2 = \frac{1}{12}(324)(10)^3$$
$$= 27 \times 10^3 \quad mm^4$$

$$I_3 = I_1 = 23{,}438 \quad mm^4$$

$$I_y = I_1 + I_2 + I_3 = 46.903 \times 10^6 \; mm^4 = 46.903 \times 10^{-6} \; m^4$$

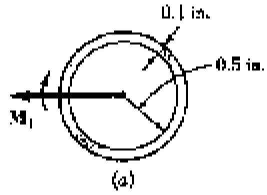
$$c = \frac{250}{2} \; mm = 125 \; mm = 0.125 \; m$$

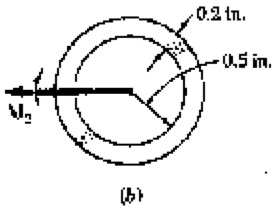
$$\sigma_{all} = \frac{\sigma_u}{F.S.} = \frac{450 \times 10^6}{3.0} = 150 \times 10^6 \; Pa$$

$$\sigma = \frac{Mc}{I} \qquad M_y = \frac{\sigma_{all} I}{c} = \frac{(150 \times 10^6)(46.903 \times 10^{-6})}{0.125}$$

$$= 56.3 \times 10^3 \; N \cdot m = 56.3 \; kN \cdot m$$

PROBLEM 4.5





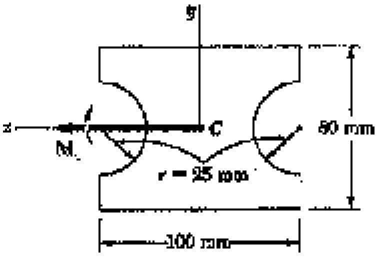
4.5 Using an allowable stress of 16 ksi, determine the largest that can be applied to each pipe.

SOLUTION

$$(a) \quad I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(0.6^4 - 0.5^4) = 52.7 \times 10^{-3} \; in^4$$
$$c = 0.6 \; in$$
$$\sigma = \frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma I}{c} = \frac{(16)(52.7 \times 10^{-3})}{0.6}$$
$$= 1.405 \; kip \cdot in$$

$$(b) \quad I = \frac{\pi}{4}(0.7^4 - 0.5^4) = 139.49 \times 10^{-3} \; in^4$$
$$c = 0.7 \; in$$
$$\sigma = \frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma I}{c} = \frac{(16)(139.49 \times 10^{-3})}{0.7}$$
$$= 3.19 \; kip \cdot in$$

PROBLEM 4.6



4.6 A nylon spacing bar has the cross section shown. Knowing that the allowable stress for the grade of nylon used is 24 MPa, determine the largest couple M_z that can be applied to the bar.

SOLUTION

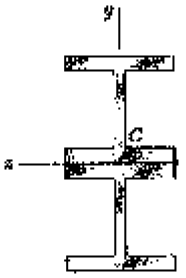
$$I = I_{rect} - I_{circle}$$
$$= \frac{1}{12}bh^3 - \frac{\pi}{4}r^4$$
$$= \frac{1}{12}(100)(80)^3 - \frac{\pi}{4}(25)^4 = 3.9599\times10^6 \text{ mm}^4$$
$$= 3.9599\times10^{-6} \text{ m}^4$$

$$c = \frac{80}{2} = 40 \text{ mm} = 0.040 \text{ m}$$

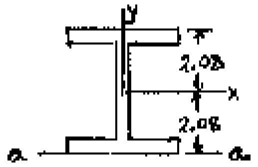
$$\sigma = \frac{Mc}{I} \qquad M = \frac{\sigma I}{c} = \frac{(24\times10^6)(3.9599\times10^{-6})}{0.040} = 2.38\times10^3 \text{ N}\cdot\text{m}$$
$$= 2.38 \text{ kN}\cdot\text{m}$$

PROBLEM 4.7

4.7 and 4.8 Two W 4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.



SOLUTION



Properties of W 4×13 rolled section
See Appendix B

Area = 3.83 in² Depth = 4.16 in
I_x = 11.3 in⁴

For one rolled section, moment of inertia about axis a-a is

$$I_a = I_x + Ad^2 = 11.3 + (3.83)(2.08)^2 = 27.87 \text{ in}^4$$

For both sections $I_z = 2I_a = 55.74$ in⁴

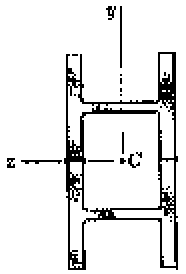
c = depth = 4.16 in

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

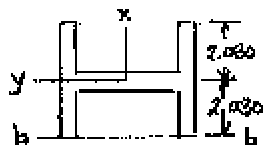
$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(55.74)}{4.16} = 259 \text{ kip}\cdot\text{in}$$

PROBLEM 4.8

4.7 and 4.8 Two W 4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.



SOLUTION



Properties of W 4×13 rolled section
See Appendix B

Area = 3.83 in² Width = 4.060 in
I_y = 3.86 in⁴

For one rolled section, moment of inertia about axis b-b is

$$I_b = I_y + Ad^2 = 3.86 + (3.83)(2.030)^2 = 19.643 \text{ in}^4$$

For both sections $I_z = 2I_b = 39.286$ in⁴

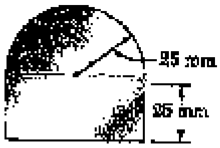
c = width = 4.060 in

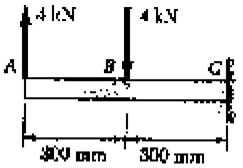
$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(39.286)}{4.060} = 187.1 \text{ kip}\cdot\text{in}$$

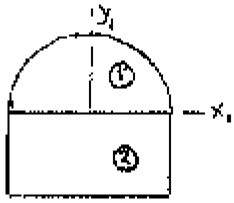
PROBLEM 4.9

4.9 through 4.11 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.





SOLUTION



$A_1 = \frac{\pi}{2} r^2 = \frac{\pi}{2}(25)^2 = 981.7 \text{ mm}^2$

$\bar{y}_1 = \frac{4r}{3\pi} = \frac{(4)(25)}{3\pi} = 10.610 \text{ mm}$

$A_2 = bh = (50)(25) = 1250 \text{ mm}^2$

$\bar{y}_2 = -\frac{h}{2} = -\frac{25}{2} = -12.5 \text{ mm}$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = \frac{(981.7)(10.610) + (1250)(-12.5)}{981.7 + 1250}$$

$= -2.334 \text{ mm}$

$\bar{I}_1 = I_{x_1} - A_1\bar{y}_1^2 = \frac{\pi}{8}r^4 - A_1\bar{y}_1^2 = \frac{\pi}{8}(25)^4 - (981.7)(10.610)^2 = 42.886 \times 10^3 \text{ mm}^4$

$d_1 = \bar{y}_1 - \bar{y} = 10.610 - (-2.334) = 12.944 \text{ mm}$

$I_1 = \bar{I}_1 + A_1 d_1^2 = 42.866 \times 10^3 + (981.7)(12.944)^2 = 207.35 \times 10^3 \text{ mm}^4$

$\bar{I}_2 = \frac{1}{12}bh^3 = \frac{1}{12}(50)(25)^3 = 65.104 \times 10^3 \text{ mm}^4$

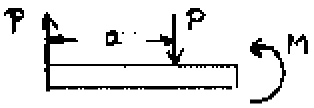
$d_2 = |\bar{y}_2 - \bar{y}| = |-12.5 - (-2.334)| = 10.166 \text{ mm}$

$I_2 = \bar{I}_2 + A_2 d_2^2 = 65.104 \times 10^3 + (1250)(10.166)^2 = 194.288 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 401.16 \times 10^3 \text{ mm}^4 = 401.16 \times 10^{-9} \text{ m}^4$

$y_{top} = 25 + 2.334 = 27.334 \text{ mm} = 0.027334 \text{ m}$

$y_{bot} = -25 + 2.334 = -22.666 \text{ mm} = -0.022666 \text{ m}$



$M - Pa = 0 \qquad M = Pa = (4 \times 10^3)(300 \times 10^{-3})$
$= 1200 \text{ N·m}$

$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(1200)(0.027334)}{401.16 \times 10^{-9}} = -81.76 \times 10^6 \text{ Pa}$$

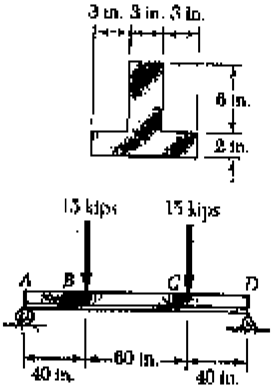
$= -81.8 \text{ MPa}$ ◀

$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(1200)(-0.022666)}{401.16 \times 10^{-9}} = 67.80 \times 10^6 \text{ Pa}$$

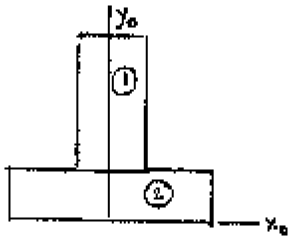
$= 67.8 \text{ MPa}$ ◀

PROBLEM 4.10

4.9 through 4.11 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.



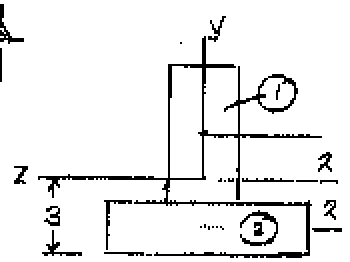
SOLUTION



	A	$\bar{y}_o$	$A\bar{y}_o$
1	18	5	90
2	18	1	18
Σ	36		108

$$\bar{Y}_o = \frac{108}{36} = 3 \text{ in}$$

Neutral axis lies 3 in. above the base.

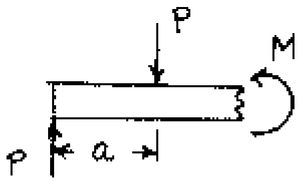


$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$

$$y_{top} = 5 \text{ in} \qquad y_{bot} = -3 \text{ in.}$$



$$M - Pa = 0$$

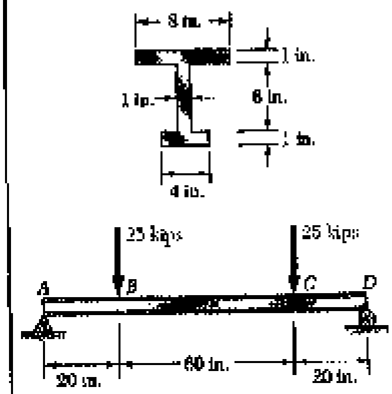
$$M = Pa = (15)(40) = 600 \text{ kip}\cdot\text{in.}$$

$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(600)(5)}{204} = -14.71 \text{ ksi}$$

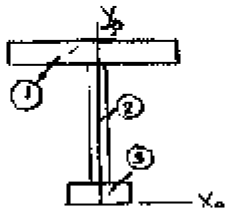
$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(600)(-3)}{204} = 8.82 \text{ ksi}$$

PROBLEM 4.11

4.9 through *4.11* Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.



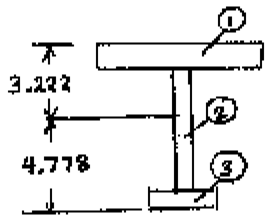
SOLUTION



	A	$\bar{y}_o$	$A\bar{y}_o$
1	8	7.5	60
2	6	4	24
3	4	0.5	2
Σ	18		86

$$\bar{Y}_o = \frac{86}{18} = 4.778 \text{ in}$$

Neutral axis lies 4.778 in above the base.



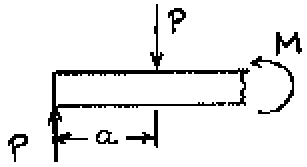
$$I_1 = \tfrac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \tfrac{1}{12}(8)(1)^3 + (8)(2.722)^2$$
$$= 59.94 \text{ in}^4$$

$$I_2 = \tfrac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \tfrac{1}{12}(1)(6)^3 + (6)(0.778)^2$$
$$= 21.63 \text{ in}^4$$

$$I_3 = \tfrac{1}{12} b_3 h_3^3 + A_3 d_3^2 = \tfrac{1}{12}(4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.54 = 156.16 \text{ in}^4$$

$y_{top} = 3.222$ in $\qquad y_{bot} = -4.778$ in



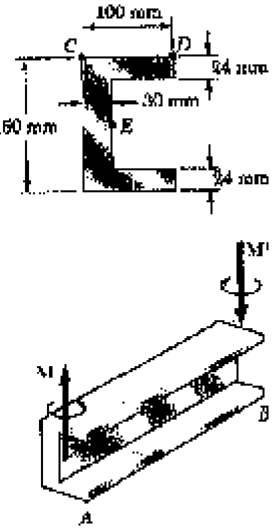
$$M - Pa = 0$$
$$M = Pa = (25)(20) = 500 \text{ kip}\cdot\text{in}$$

$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(500)(3.222)}{155.16} = -10.38 \text{ ksi}$$

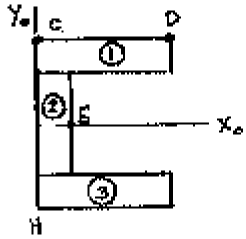
$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(500)(-4.778)}{155.16} = 15.40 \text{ ksi}$$

PROBLEM 4.12

4.12 Two equal and opposite couples of magnitude of $M = 15$ kN·m are applied to the channel-shaped beam AB. Observing that the couples cause the beam to bend in a horizontal plane, determine the stress (*a*) at point C, (*b*) at point D, (*c*) at point E.

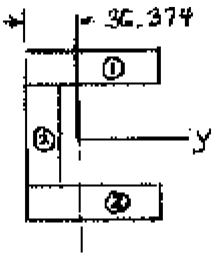


SOLUTION



	A, mm^2	$\bar{x}_o, mm$	$A\bar{x}_o, mm^3$
①	2400	50	120×10^3
②	3060	15	45.9×10^3
③	2400	50	120×10^3
Σ	7860		285.9×10^3

$$\bar{X} = \frac{285.9 \times 10^3}{7860} = 36.374 \text{ mm}$$



$y_C = -36.374$ mm $= -0.036374$ m

$y_D = 100 - 36.374 = 63.626$ mm $= 0.63626$ m

$y_E = 30 - 36.374 = -6.374$ mm $= -0.006374$ m

$d_1 = 50 - 36.374 = 13.626$ mm
$d_2 = 36.374 - 15 = 21.374$ mm
$d_3 = d_1$

$$I_1 = I_3 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(24)(100)^3 + (2400)(13.626)^2 = 2.4456 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(102)(30)^3 + (3060)(21.374)^2 = 1.6275 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 6.5187 \times 10^6 \text{ mm}^4 = 6.5187 \times 10^{-6} \text{ m}^4$$

$$M = 15 \times 10^3 \text{ N·m}$$

(a) Point C: $\sigma_C = -\frac{M y_C}{I} = -\frac{(15 \times 10^3)(-0.036374)}{6.5187 \times 10^{-6}} = 83.7 \times 10^6$ Pa $= 83.7$ MPa ◄

(b) Point D: $\sigma_D = -\frac{M y_D}{I} = -\frac{(15 \times 10^3)(0.063626)}{6.5187 \times 10^{-6}} = -146.4 \times 10^6$ Pa $= -146.4$ MPa ◄

(c) Point E: $\sigma_E = -\frac{M y_E}{I} = -\frac{(15 \times 10^3)(-0.006374)}{6.5187 \times 10^{-6}} = 14.67 \times 10^6$ Pa $= 14.67$ MPa ◄